1 Consider a quadratic equation $ax^2 + 2bx + c = 0$, where a, b and c are positive real numbers. If the equation has no real roots, then which of the following is true?

(A) a, b, c cannot be in AP or HP, but can be in GP.

(B) a, b, c cannot be in GP or HP, but can be in AP.

(C) a, b, c cannot be in AP or GP, but can be in HP.

(D) a, b, c cannot be in AP, GP or HP.

2 A unit square has its corners chopped off to form a regular polygon with eight sides. What is the area of this polygon?

(A) $2(\sqrt{3} - \sqrt{2})$ (B) $2\sqrt{2} - 2$ (C) $\frac{\sqrt{2}}{2}$ (D) $\frac{7}{9}$.

3 A solid cube of side five centimeters has all its faces painted. The cube is sliced into smaller cubes, each of side one centimeter. How many of these smaller cubes will have paint on exactly one of its faces?

(A) 25 (B) 54 (C) 126 (D) 150.

4 Let z be a complex number such that $\dfrac{z-i}{z-1}$ is purely imaginary. Then the minimum value of $|z - (2+2i)|$ is

(A) $2\sqrt{2}$ (B) $\sqrt{2}$ (C) $\dfrac{3}{\sqrt{2}}$ (D) $\dfrac{1}{\sqrt{2}}$.

5 Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function such that for any two real numbers x and y,

$$|f(x) - f(y)| \leq 7\,|x - y|^{201}.$$

Then,

(A) $f(101) = f(202) + 8$ (B) $f(101) = f(201) + 1$

(C) $f(101) = f(200) + 2$ (D) None of the above.

6 In the Mathematics department of a college, there are 60 first year students, 84 second year students, and 108 third year students. All of these students are to be divided into project groups such that each group has the same number of first year students, the same number of second year students, and the same number of third year students. What is the smallest possible size of each group?

(A) 9 (B) 12 (C) 19 (D) 21.

7 Let a, b and c be real numbers, each greater than 1, such that

$$\frac{2}{3}\log_b a + \frac{3}{5}\log_c b + \frac{5}{2}\log_a c = 3.$$

If the value of b is 9, then the value of a must be

(A) $\sqrt[3]{81}$ (B) $\frac{27}{2}$ (C) 18 (D) 27.

8 Consider a triangle ABC. The sides AB and AC are extended to points D and E, respectively, such that $AD = 3AB$ and $AE = 3AC$. Then one diagonal of $BDEC$ divides the other diagonal in the ratio

(A) $1 : 3$ (B) $1 : \sqrt{3}$ (C) $1 : 2$ (D) $1 : \sqrt{2}$.

9 The area of the region bounded by the curve $y = \tan x$, the x-axis and the tangent to the curve $y = \tan x$ at $x = \frac{\pi}{4}$ is

(A) $\log_e 2 - \frac{1}{2}$ (B) $\log_e 2 + \frac{1}{2}$ (C) $\frac{1}{2}(\log_e 2 - \frac{1}{2})$ (D) $\frac{1}{2}(\log_e 2 + \frac{1}{2})$.

10 Let V be the set of vertices of a regular polygon with twenty sides. Three distinct vertices are chosen at random from V. Then, the probability that the chosen triplet are the vertices of a right angled triangle is

(A) $\frac{7}{19}$ (B) $\frac{3}{19}$ (C) $\frac{3}{38}$ (D) $\frac{1}{38}$.

11 A "basic row operation" on a matrix means adding a multiple of one row to another row. Consider the matrices

$$A = \begin{pmatrix} x & 5 & x \\ 1 & 3 & -2 \\ -2 & -2 & 2 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 0 & 0 & 21 \\ 1 & -1 & -14 \\ 0 & \frac{4}{3} & 4 \end{pmatrix}$$

It is given that B can be obtained from A by applying finitely many basic row operations. Then, the value of x is:

(A) 3 (B) -3 (C) -1 (D) 2.

12 Let C be a circle of area A with centre at O. Let P be a moving point such that its distance from O is always twice the length of a tangent drawn from P to the circle. Then the point P must move along

(A) the sides of a square centred at O, with area $\frac{4}{3}A$.

(B) the sides of an equilateral triangle centred at O, with area $4A$.

(C) a circle centred at O, with area $\frac{4}{3}A$.

(D) a circle centred at O, with area $4A$.

13 A moving line intersects the lines $x+y=0$ and $x-y=0$ at the points A and B such that the area of the triangle with vertices $(0,0)$, A and B has a constant area C. The locus of the midpoint of AB is given by the equation

(A) $(x^2+y^2)^2=C^2$ (B) $(x^2-y^2)^2=C^2$
(C) $(x+y)^2=C^2$ (D) $(x-y)^2=C^2$.

14 Let $A=\{1,2,3,4,5,6\}$ and $B=\{a,b,c,d,e\}$. How many functions $f: A\to B$ are there such that for every $x\in A$, there is one and exactly one $y\in A$ with $y\neq x$ and $f(x)=f(y)$?

(A) 450 (B) 540 (C) 900 (D) 5400.

15 Two persons, both of height h, are standing at a distance of h from each other. The shadow of one person cast by a vertical lamp-post placed between the two persons is double the length of the shadow of the other. If the sum of the lengths of the shadows is h, then the height of the lamp post is

(A) $\frac{\sqrt{3}}{2}h$ (B) $2h$ (C) $\left(\frac{1+\sqrt{2}}{2}\right)h$ (D) $\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)h$.

16 Let S be the set of all points z in the complex plane such that

$$\left(1+\frac{1}{z}\right)^4=1.$$

Then, the points of S are

(A) vertices of a rectangle
(B) vertices of a right-angled triangle
(C) vertices of an equilateral triangle
(D) collinear

17 A circular lawn of diameter 20 meters on a horizontal plane is to be illuminated by a light-source placed vertically above the centre of the lawn. It is known that the illuminance at a point P on the lawn is given by the formula $I=\frac{C\sin\theta}{d^2}$ for some constant C, where d is the distance of P from the light-source and θ is the angle between the line joining the centre of the lawn to P and the line joining the light-source to P. Then the maximum possible illuminance at a point on the circumference of the lawn is

(A) $\frac{C}{75\sqrt{3}}$ (B) $\frac{C}{100\sqrt{3}}$ (C) $\frac{C}{150\sqrt{3}}$ (D) $\frac{C}{250\sqrt{3}}$.

18 Let f and g be two real-valued continuous functions defined on the closed interval $[a, b]$, such that $f(a) < g(a)$ and $f(b) > g(b)$. Then the area enclosed between the graphs of the two functions and the lines $x = a$ and $x = b$ is always given by

(A) $\int_a^b |f(x) - g(x)|\, dx$ (B) $\left|\int_a^b (f(x) - g(x))dx\right|$

(C) $\left|\int_a^b (|f(x)| - |g(x)|)dx\right|$ (D) $\int_a^b ||f(x)| - |g(x)||\, dx$.

19 Consider the function $f : \mathbb{R} \longrightarrow \mathbb{R}$, defined as follows:

$$f(x) = \begin{cases} (x-1)\min\left\{x, x^2\right\} & \text{if } x \geq 0 \\ x \min\left\{x, \dfrac{1}{x}\right\} & \text{if } x < 0 \end{cases}$$

Then, f is

(A) differentiable everywhere.

(B) *not* differentiable at exactly one point.

(C) *not* differentiable at exactly two points.

(D) *not* differentiable at exactly three points.

20 Let $f : [0, 2] \to \mathbb{R}$ be a continuous function such that

$$\frac{1}{2}\int_0^2 f(x)dx < f(2).$$

Then which of the following statements must be true?

(A) f must be strictly increasing.

(B) f must attain a maximum value at $x = 2$.

(C) f cannot have a minimum at $x = 2$.

(D) None of the above.

21 In a triangle ABC, $3\sin A + 4\cos B = 6$ and $4\sin B + 3\cos A = 1$ hold. Then the angle C equals

(A) $30°$ (B) $60°$ (C) $120°$ (D) $150°$.

22 Let $\theta = \frac{2\pi}{7}$ and consider the following matrix

$$A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

If A^n means $A \times \cdots \times A$ (n times), then A^{100} is

(A) $\begin{pmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}$ (B) $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

(C) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (D) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.

23 Consider all the permutations of the twenty six English letters that start with z. In how many of these permutations the number of letters between z and y is less than those between y and x?

(A) $6 \times 23!$ (B) $6 \times 24!$ (C) $156 \times 23!$ (D) $156 \times 24!$.

24 Let $P = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ and $Q = \left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ be two vertices of a regular polygon having 12 sides such that PQ is a diameter of the circle circumscribing the polygon. Which of the following points is not a vertex of this polygon?

(A) $\left(\frac{\sqrt{3}-1}{2\sqrt{2}}, \frac{\sqrt{3}+1}{2\sqrt{2}}\right)$ (B) $\left(\frac{\sqrt{3}+1}{2\sqrt{2}}, \frac{\sqrt{3}-1}{2\sqrt{2}}\right)$

(C) $\left(\frac{\sqrt{3}+1}{2\sqrt{2}}, \frac{1-\sqrt{3}}{2\sqrt{2}}\right)$ (D) $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.

25 Let a, b, c be real numbers such that $a = a^2 + b^2 + c^2$. What is the smallest possible value of b?

(A) 0 (B) -1 (C) $-\frac{1}{4}$ (D) $-\frac{1}{2}$.

26 Consider the function $f : \mathbb{R} \to \mathbb{R}$ defined as

$$f(x) = \begin{cases} \dfrac{x}{e^x - 1} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$$

Then which one of the following statements is correct?

(A) f is not continuous at $x = 0$.

(B) f is continuous but not differentiable at $x = 0$.

(C) f is differentiable at $x = 0$ and $f'(0) = -\frac{1}{2}$.

(D) f is differentiable at $x = 0$ and $f'(0) = \frac{1}{2}$.

27 Let the function $f : [0, 1] \to \mathbb{R}$ be defined as

$$f(x) = \max\left\{\frac{|x - y|}{x + y + 1} : 0 \leq y \leq 1\right\} \text{ for } 0 \leq x \leq 1.$$

Then which of the following statements is correct?

(A) f is strictly increasing on $\left[0, \frac{1}{2}\right]$ and strictly decreasing on $\left[\frac{1}{2}, 1\right]$.

(B) f is strictly decreasing on $\left[0, \frac{1}{2}\right]$ and strictly increasing on $\left[\frac{1}{2}, 1\right]$.

(C) f is strictly increasing on $\left[0, \frac{\sqrt{3}-1}{2}\right]$ and strictly decreasing on $\left[\frac{\sqrt{3}-1}{2}, 1\right]$.

(D) f is strictly decreasing on $\left[0, \frac{\sqrt{3}-1}{2}\right]$ and strictly increasing on $\left[\frac{\sqrt{3}-1}{2}, 1\right]$.

28 For a positive real number α, let S_α denote the set of points (x, y) satisfying

$$|x|^\alpha + |y|^\alpha = 1.$$

A positive number α is said to be *good* if the points in S_α that are closest to the origin lie only on the coordinate axes. Then

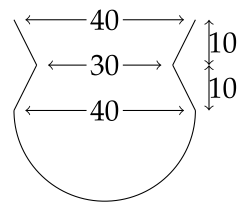
(A) all α in $(0, 1)$ are good and others are not good.

(B) all α in $(1, 2)$ are good and others are not good.

(C) all $\alpha > 2$ are good and others are not good.

(D) all $\alpha > 1$ are good and others are not good.

29 A water pitcher has a hemispherical bottom and a neck in the shape of two truncated cones of the same size. The vertical cross-section of the pitcher with relevant dimensions is shown in the figure. Suppose that the pitcher is filled with water to the brim. If a solid cylinder with diameter 24 cm and height greater than 60 cm is inserted vertically into the pitcher as far down to the bottom as possible, how much water would remain in the pitcher?



(A) 6316π cm^3 (B) 6116π cm^3 (C) 6336π cm^3 (D) 6136π cm^3

30 Let $f : [-1, 1] \to \mathbb{R}$ be a function such that $f\left(\sin \dfrac{x}{2}\right) = \sin x + \cos x$, for all $x \in [-\pi, \pi]$. The value of $f\left(\frac{3}{5}\right)$ is

(A) $\frac{24}{25}$ (B) $\frac{31}{25}$ (C) $\frac{33}{25}$ (D) $\frac{7}{5}$.